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Dirac Neutrinos and Hybrid Inflation from String Theory

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Abstract

We consider a possible scenario for the generation of Dirac neutrino masses motivated by Type I string theory. The smallness of the neutrino Yukawa couplings is explained by an anisotropic compactification with one compactification radius larger than the others. In addition to this we utilise small Yukawa couplings to develop strong links between the origin of neutrino masses and the physics driving inflation. We construct a minimal model which simultaneously accommodates small Dirac neutrino masses leading to bi-large lepton mixing as well as an inflationary solution to the strong CP and to the μ problem.

1 Introduction

The evidence of neutrino masses is the first clear signal of physics beyond the Standard Model (SM). The most popular explanation of neutrino masses and of their smallness is the well known see-saw mechanism, where heavy SM-singlet right-handed neutrinos are introduced. Another possibility, even more minimal than the see-saw mechanism in the sense that it has less free parameters, is that neutrinos have pure Dirac masses $m^{\nu}_{\mathrm{LR}} = Y_\nu v_{\mathrm{u}}$, generated from a Yukawa coupling

$$(Y_\nu)_{ij} \overline{L}_i H_{\mathrm{u}} \nu^{\mathrm{c}}_{\mathrm{R}j} \ . \tag{1}$$

Obviously, generating neutrino masses of order 0.1 eV requires Yukawa couplings $(Y_\nu)_{ij} \sim 10^{-12}$. This required smallness is the main objection against Dirac neutrinos - and we are going to address it in this letter. Existing explanations of this smallness utilise for instance right-handed singlet neutrinos propagating in the bulk, or allow only highly suppressed effective operators, e.g. by linking the smallness of neutrino masses to supersymmetry breaking [1, 2]. Heterotic string constructions can also lead to Dirac neutrinos and in some classes of Heterotic orbifolds Dirac neutrino masses may even be more favoured than the see-saw mechanism [3].

Small couplings are also required for the inflationary solution to the strong CP and to the μ problem of the MSSM, proposed in [4, 5]. The μ-term arises from a superpotential term

$$\lambda \phi H_{\mathrm{u}} H_{\mathrm{d}} \tag{2}$$

within the model of inflation proposed in [5]. The vev $\langle \phi \rangle$ of the inflaton after inflation generates $\mu = \lambda \langle \phi \rangle$ and furthermore breaks Peccei-Quinn symmetry solving the strong CP problem. Satisfying the constraints on the axion decay constant and the scale of μ requires $\langle \phi \rangle = 10^{13}$ GeV and a small coupling λ of order 10^{-10}.

From the above discussion, it is clear that very small Yukawa couplings are a prerequisite for both Dirac neutrinos and for the inflationary solution to the strong CP and μ problem. It should also be noted that both the right-handed neutrinos ν_{Ri} and the inflaton ϕ are special in the sense that they should have only small couplings to ordinary matter and that they are effectively SM-singlets. In [6] it has been shown how such small couplings might arise in the context of Type I string theory. The details are reviewed in Appendix A. The mechanism has the simple graphical illustration shown in Fig. 1. Matter corresponds to open string states with ends confined either to one of the three orthogonal stacks of D5-branes or to the stack of space-filling D9-branes. With a compactification radius for one of the D5-branes[1] ($\mathrm{D5}_1$ in Fig. 1) being larger than the compactification radius for the others it can be shown that the gauge and Yukawa couplings associated with this brane are small. Fields corresponding to open strings confined to $\mathrm{D5}_2$ branes with both ends can only participate in interactions with small couplings and it is some of these fields that we will identify with the right-handed neutrinos. The $\mathrm{D5}_2$ branes wrap with a smaller radius and hence have a gauge coupling large enough to be associated with the Standard Model. As a result the Standard Model gauge group must arise from this stack of branes.

In this letter, we construct a minimal string-motivated model which simultaneously accommodates small Dirac neutrino masses leading to bi-large lepton mixing as well as the inflationary solution [5] to the strong CP and to the μ problem.

By string motivated, we mean that we take some restrictions from string theory, i.e. so-called string selection rules, and construct a field theory model consistent with these rules. We do not claim that an embedding of our particular model in string theory is possible. In fact, we do expect that there will emerge additional constraints if one attempts to embed our scenario in string theory. Nevertheless, we find it useful

[1] The branes wrap the compact dimensions hence they can be associated with their radii.

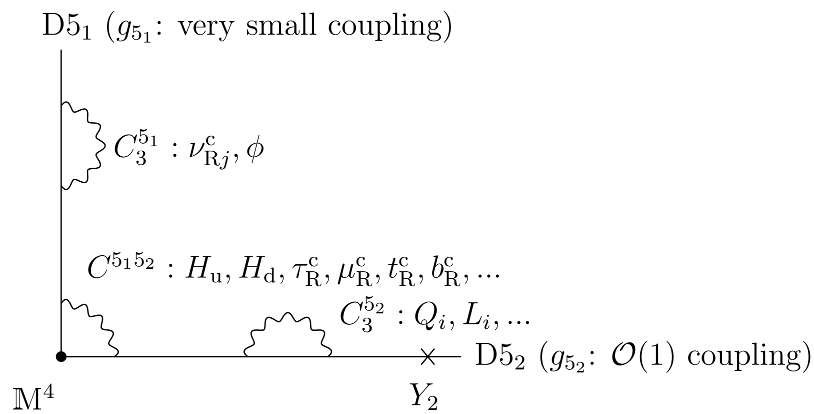

Figure 1: Graphical illustration of the origin of small couplings in our scenario. Chiral matter corresponds to open string states C with ends confined either to one of the three orthogonal stacks of D5-branes or to the stack of space-filling D9-branes. While the gauge coupling associated with the stack of branes D5_2 is $\mathcal{O}(1)$, the gauge coupling associated with D5_1 is $\mathcal{O}(10^{-10})$. Fields assigned to states $C_3^{5_1}$ can only participate in interactions with small couplings. The stacks of branes overlap in Minkowski space $\mathbb{M}^4$, but are orthogonal in the compactified dimensions. Y_2 is a twisted modulus localised within the extra dimensions, but free to move in Minkowski space.

to identify attractive routes for explaining the smallness of Dirac neutrino masses within the framework of string theory from a bottom-up perspective.

The layout of the rest of the paper is as follows. Section 2 provides a brief discussion of the string framework we consider in this paper to clarify its use in model building. This is followed by the main body of the paper, section 3, in which we detail the model, its construction, the masses and MNS mixings. Having discussed the lepton sector section 4 considers the inflation model that could be realised within the same framework. The conclusions are to be found in section 5 and are followed by two appendices. In Appendix A we present a more thorough account of the string framework and we clarify our lepton mixing conventions in Appendix B.

2 Restrictions and benefits from string selection rules

Let us briefly state how the scenario outlined in the introduction and illustrated in Fig. 1 restricts model building. For the details we refer the reader to Appendix A. In

particular we would like to draw the reader's attention to Eq. (A.3) from which we extract the following terms to be utilised in the model:

$$W = g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2} + g_{5_2} C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2} \, . \tag{3}$$

$C_3^{5_1}$, $C_3^{5_2}$ and $C^{5_1 5_2}$ are low energy excitations of strings: charged chiral superfields. The superscripts denote the branes which the strings end on and terms with different subscripts transform differently under the gauge group associated with the brane. The string construction involves assigning fields to low energy string excitations (the C terms) and showing that only the gauge invariant operators appearing in Eq. (A.3) can be written down. For the purpose of our model we will make use of the fact that the only couplings of $C_3^{5_1}$ and $C_3^{5_2}$ to intersection states $C^{5_1 5_2}$ appearing in Eq. (A.3) are those of Eq. (3). The C terms can have more than one field assigned to them and the fields can only transform under the gauge groups of the branes to which the C field attaches. For example the $C^{5_1 5_2}$ term has fields that transform under the gauge groups of the $D5_1$ and $D5_2$ branes. If the operator does not appear in Eq. (A.3) or cannot transform under gauge groups required by the field theory then we say such an operator is forbidden by the string selection rules. For more details about the rules and the explicit construction we refer the reader to [6] and merely quote the results here. The gauge couplings $g_{5_1}, g_{5_2}, g_{5_3}$ and g_9 on the branes are related to the Planck scale M_p and the string scale M_* by

$$g_{5_1} g_{5_2} g_{5_3} g_9 = 32\pi^2 \left(\frac{M_*}{M_p} \right)^2 . \tag{4}$$

Here we shall choose the couplings $g_{5_2} = \sqrt{\frac{4\pi}{24}}$ (to give $\alpha_{\mathrm{GUT}} = 1/24$, consistent with gauge coupling unification), $g_{5_3} = g_9 = 2$ and $g_{5_1} = 10^{-10}$, which results in the string scale $M_* = 10^{13}$ GeV.[2] The gauge couplings on the branes are related to the extra-

[2]Gauge coupling unification at the string scale, $M_* = 10^{13}$, where the gauge coupling on the brane, g_{5_2}, is specified, is possible due to power law running. The requirement of gauge coupling unification would constrain the matter content and mass spectrum of complete models - which is however beyond the scope of this letter. In the presence of twisted moduli, this constraint would be modified.

dimensional radii as $g_{5_i}^2 = 2\pi\lambda_{\rm I}/(R_i^2 M_*^2)$, $g_9^2 = 2\pi\lambda_{\rm I}/(R_1^2R_2^2R_3^2M_*^6)$, where $\lambda_{\rm I}$ is the ten dimensional dilaton that governs the strength of string interactions.

Thus, the geometry of the compactification determines the couplings of the theory. Furthermore, assigning the superfields of a model to open string excitations restricts the possible superpotential couplings to the ones given in Eq. (A.3).

3 Dirac neutrinos

With the right-handed neutrino fields $\nu^{\rm c}_{{\rm R}i}$ assigned to string states $C_3^{5_1}$ and with the gauge coupling g_{5_1} on the branes $\mathrm{D}5_1$ being of order 10^{-10} (the choice $g_{5_1} = \mathcal{O}(10^{-10})$ is motivated by the hybrid inflation model which we will discuss in Sec. 4) light neutrinos of Dirac-type naturally emerge - all couplings to $\nu^{\rm c}_{{\rm R}i}$ are suppressed by, at least, g_{5_1}. On the other hand, right-handed charged leptons assigned to string states $C_3^{5_2}$ corresponds to the larger charged lepton masses since the gauge coupling g_{5_2} on the branes $\mathrm{D}5_2$ is of order 1.

Let us now consider the generation of lepton masses in more detail. With $H_{\rm u}$ and the right-handed tau $\tau^{\rm c}_{\rm R}$ assigned to intersection states $C^{5_15_2}$ and the lepton doublet assigned to a string state $C_3^{5_2}$, we see from Eq. (3) that a renormalizable Yukawa coupling $y_\tau \sim g_{5_2} = \mathcal{O}(1)$

$$g_{5_2} C_3^{5_2} C^{5_15_2} C^{5_15_2} \quad \rightarrow \quad g_{5_2} L_i H_{\rm d} \tau^{\rm c}_{\rm R} \quad (\mathcal{O}(1) \text{ coupling } g_{5_2}) \tag{5}$$

is allowed by the string selection rules. As we have seen, Yukawa couplings in this string motivated setup cannot be chosen freely, but are fixed by the values of gauge couplings. The $\mathcal{O}(1)$-coupling to $\tau^{\rm c}_{\rm R}$ of Eq. (5) can be consistent with low energy experimental data for appropriately chosen large $\tan\beta$.[3] However, the Yukawa cou-

[3] Note that if 3rd family quark Yukawa couplings also stem from analogous renormalizable couplings, 3rd family gauge-Yukawa unification $y_\tau \sim y_b \sim y_t \sim g_{5_2}$ for the SM fermions holds, up to order 1 coefficients, even though they are not unified in an irrep of a unified gauge group. We will not address the details of the quark sector in this letter.

plings leading e.g. to the mass m_μ of the muon, though allowed by the string selection rules, do not stem from an analogous term to Eq. (5) since they will be forbidden by the symmetries of the model discussed below. Our strategy is to obtain it from a non-renormalizable operator generated via a supersymmetric Froggatt-Nielsen (FN) [7] mechanism, for example

$$\frac{g_{5_2}}{\langle\psi\rangle}\,\langle A\rangle\, L_2 H_{\rm d}\mu^{\rm c}_{\rm R}\ . \tag{6}$$

This is represented in Fig. 2. Such Froggatt-Nielsen supergraphs are allowed by the string selection rules if the messenger fields are assigned to intersection states and if the masses for the messenger fields stem from the vev $\langle\psi\rangle$ of the scalar component of an additional field ψ, assigned to a string state $C_3^{5_2}$. The flavon field A has to be assigned to a string state $C_3^{5_2}$ as well and the muon mass from this operator is suppressed by $\langle A\rangle\,/\,\langle\psi\rangle$. Generating the muon mass of the right order requires $\langle A\rangle\,/\,\langle\psi\rangle = \mathcal{O}(10^{-3})$.[4] We have ignored the electron mass here, which can be generated by higher-dimensional operators in a straightforward way.

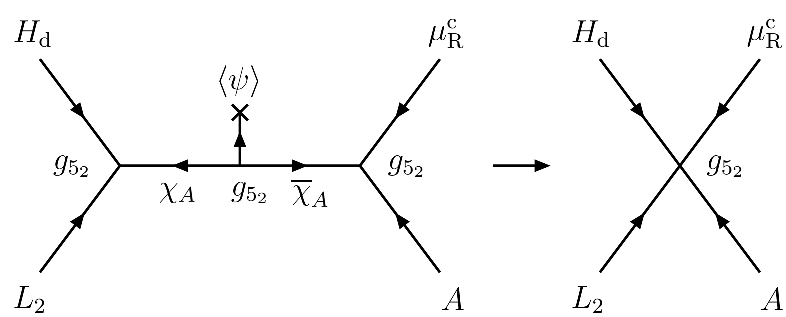

Figure 2: Froggatt Nielsen supergraphs leading to the muon mass. Higher-dimensional FN diagrams can generate NNLO Yukawa couplings, e.g. for realising the electron mass.

Let us now consider the neutrino sector: since atmospheric neutrino oscillations suggest a neutrino mass scale $m_3 \approx \sqrt{\Delta m^2_{31}} \approx 0.05$ eV [8], a Yukawa coupling

[4]Note that we do not have to specify the values of the vevs at this stage. We should however keep in mind that that $\langle\psi\rangle$, i.e. the mass of the messenger fields in the FN mechanism, should be sufficiently below the masses of the winding modes of the messenger fields (in our scenario $\approx n10^8$ GeV, see Appendix A), so that the effects of the extra dimensions are under control.

$(Y_\nu)_{ij} \approx 10^{-12} - 10^{-13}$ is required for generating the largest neutrino mass eigenvalue m_3. Thus, a renormalizable Yukawa coupling $(Y_\nu)_{ij} = g_{5_1} \sim 10^{-10}$ would be already in the right range, but still somewhat too large. In addition to $m_3 \approx 0.05$ eV the experimental results for Δm^2_{21} require $m_2 \approx \sqrt{\Delta m^2_{21}} \approx 0.01$ eV, which also requires a suppression of about 10^{-3} compared to g_{5_1}.

In fact, we see that the string selection rules forbid the renormalizable tree level Yukawa couplings involving $\nu^{\mathrm{c}}_{\mathrm{R}j}$ and L_i. However, as in the charged lepton sector for m_μ, we can rely on a FN mechanism (cf. Fig. 3) for obtaining the neutrino Yukawa couplings which can then also have the desired additional suppression. The neutrino Yukawa couplings can then stem from the leading order effective operators

$$\frac{g_{5_1}}{\langle\psi\rangle} \langle F_{ij}\rangle \, L_i H_{\mathrm{u}} \nu^{\mathrm{c}}_{\mathrm{R}j} \; . \tag{7}$$

From Fig. 3 and Eq. (3) we can determine appropriate string assignments of the flavons and the messenger superfields and see that the messenger fields have to be assigned to intersection states $C^{5_1 5_2}$. The flavons are found to be both intersection and single brane states. In the neutrino sector, suppression factors similar to the one for the muon mass, $\langle F_{ij}\rangle \,/\, \langle\psi\rangle = \mathcal{O}(10^{-2}) ... \mathcal{O}(10^{-3})$, are needed.

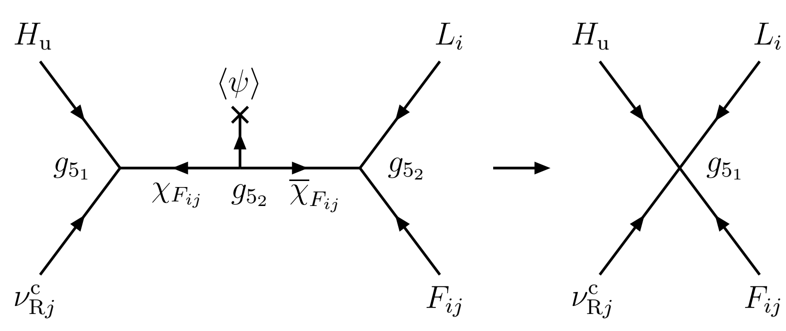

Figure 3: Froggatt Nielsen supergraphs leading to neutrino Yukawa couplings which are slightly suppressed compared to the already small tree-level value $g_{5_1} = 10^{-10}$. F_{ij} are flavon superfields and $\chi_{F_{ij}}, \bar{\chi}_{F_{ij}}$ are corresponding messenger superfields.

In summary, the Yukawa matrices of neutrinos and charged leptons could stems

from LO and NLO operators of the following form (up to $\mathcal{O}(1)$-factors):

$$Y_\nu \quad : \quad \frac{g_{5_1}}{\langle\psi\rangle} \langle F_{ij}\rangle \, L_i H_\mathrm{u} \nu^\mathrm{c}_{\mathrm{R}j} \, , \tag{8}$$

$$Y_e \quad : \quad g_{5_2} L_3 H_\mathrm{d} \tau^\mathrm{c}_\mathrm{R} + \frac{g_{5_2}}{\langle\psi\rangle} \langle A\rangle \, L_2 H_\mathrm{d} \mu^\mathrm{c}_\mathrm{R} \, . \tag{9}$$

The fact that the renormalizable Yukawa couplings $(Y_\nu)_{ij}$ are forbidden helps in three ways. Firstly, as already noted, it can lead to the desired additional suppression compared to the already small gauge coupling $g_{5_1} = \mathcal{O}(10^{-10})$. Secondly, if we use flavour symmetries for determining the structure of Y_e and Y_ν, we find that for renormalisable operators large off-diagonal elements in Y_ν would come along with identical large off-diagonal elements in Y_e - making it difficult to construct the desired large neutrino mixings. As we will see below, large lepton mixing can easily be achieved if the renormalisable coupling is forbidden. Thirdly, there is only a very mild mass hierarchy $m_3/m_2 \lesssim 5$ for m_2 and m_3 in the neutrino sector, compared 3rd and 2nd generation masses of quarks and charged leptons. This can be explained by the Yukawa couplings relevant for the neutrino masses m_3 and m_2 being generated at the same (or similar) order in the FN mechanism - which can be a consequence of the forbidden renormalizable term.

Majorana masses for the right-handed neutrinos are not allowed by a renormalisable term due to string selection rules. Higher-dimensional operators for Majorana masses are suppressed by $g^n_{5_1} \sim (10^{-10})^n$, with $n \geq 2$. For obtaining pure Dirac masses, we can impose in addition a global $\mathrm{U}(1)_{\mathrm{B-L}}$ which forbids Majorana masses for $\nu^\mathrm{c}_{\mathrm{R}i}$ to all orders.

Let us consider an explicit example with a hierarchical neutrino mass spectrum and how bi-large neutrino mixing can be realised within this scheme. We will assume that the lightest neutrino mass m_1 is approximately zero. Effectively, this means that only two right-handed neutrinos are required or equivalently that ν_{R1}'s Yukawa couplings are generated at higher order and hence heavily surpressed. For our analysis

we treat its couplings as being zero. For generating an appropriate set of operators which leads to the observed bi-large lepton mixing, we will use a $\mathbb{Z}_3$-symmetry and the $\mathrm{U}(1)_{\mathrm{R}}$-symmetry (which will be broken to matter parity) in addition to the SM gauge group $\mathrm{SU}(3)_{\mathrm{C}}\times\ \mathrm{SU}(2)_{\mathrm{L}}\times\ \mathrm{U}(1)_{\mathrm{Y}}$ which is a subgroup of the gauge group G, a copy of which is associated with the branes $\mathrm{D5}_2$.[5] The field content and the corresponding charge assignments of our minimal model is listed in table 1 and table 2, which contain the matter superfields and flavon superfields $F_{ij} \in \{a, b, c, e, f\}$ and A. They also contain the superfields of the messenger sector $\chi_{F_{ij}} \in \{\chi_a, \chi_e, \chi_A\}$, respectively. The superpotential then contains the following renormalisable terms:

$$
\begin{aligned}
W_{ren.} = \ & g_{5_2} L_3 H_{\mathrm{d}} \tau^{\mathrm{c}}_{\mathrm{R}} + g_{5_2} L_2 H_{\mathrm{d}} \chi_A + g_{5_2} \chi_A \psi \bar{\chi}_A + g_{5_2} \bar{\chi}_A \mu^{\mathrm{c}}_{\mathrm{R}} A \\
& + g_{5_1} \chi_a H_{\mathrm{u}} \nu^{\mathrm{c}}_{\mathrm{R}2} + g_{5_2} \chi_a \psi \bar{\chi}_a + g_{5_2} L_1 \bar{\chi}_a a + g_{5_2} L_2 \bar{\chi}_a b + g_{5_2} L_3 \bar{\chi}_a c \\
& + g_{5_1} \chi_e H_{\mathrm{u}} \nu^{\mathrm{c}}_{\mathrm{R}3} + g_{5_2} \chi_e \psi \bar{\chi}_e + g_{5_2} L_2 \bar{\chi}_e e + g_{5_2} L_3 \bar{\chi}_e f \\
& + g_{5_1} \phi H_{\mathrm{u}} H_{\mathrm{d}} + g_{5_1} \phi N^2 + g_{5_2} Q_3 H_{\mathrm{u}} t^{\mathrm{c}}_{\mathrm{R}} + g_{5_2} Q_3 H_{\mathrm{d}} b^{\mathrm{c}}_{\mathrm{R}} .
\end{aligned} \tag{10}
$$

From this superpotential, assuming that the flavons develop vevs, the FN diagrams of Fig. 2 and Fig. 3 lead to the mass matrices of the neutrinos and charged leptons[6]

$$
m^{\nu}_{\mathrm{LR}} \sim \left(\begin{array}{ccc} 0 & \langle a \rangle & 0 \\ 0 & \langle b \rangle & \langle e \rangle \\ 0 & \langle c \rangle & \langle f \rangle \end{array} \right) \cdot \frac{g_{5_1} \langle H_{\mathrm{u}} \rangle}{\langle \psi \rangle} , \quad m^{\mathrm{E}}_{\mathrm{LR}} \sim \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \langle A \rangle / \langle \psi \rangle & 0 \\ 0 & 0 & \mathcal{O}(1) \end{array} \right) \cdot g_{5_2} \langle H_{\mathrm{d}} \rangle \ . \tag{11}
$$

Let us now discuss neutrino masses and lepton mixing with mass matrices of the structure given in Eq. (11). We assume the sequential dominance of the Dirac neutrino Yukawa couplings

$$
\langle e \rangle , \langle f \rangle \gg \langle a \rangle , \langle b \rangle , \langle c \rangle , \tag{12}
$$

[5]The $\mathrm{U}(1)_{\mathrm{R}}$-symmetry is broken down to its $\mathbb{Z}_4$ subgroup by the appearance of gauginos' soft masses. This $\mathbb{Z}_4$ is in turn broken to its $\mathbb{Z}_2$ subgroup, matter-parity, when A obtains its vev. We require that this breaking takes place before the end of inflation to avoid the domain wall problem.

[6]It is easy to check that all the messenger fields get heavy when the flavons and ψ obtain their vevs.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_R$	$\mathbb{Z}_3$	String State
Q_3	$\mathbf{3}$	$\mathbf{2}$	1/6	-1/2	1	$C_3^{5_2}$
t_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	-2/3	-1/2	1	$C^{5_15_2}$
b_R^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	-1/2	1	$C^{5_15_2}$
H_u	$\mathbf{1}$	$\mathbf{2}$	1/2	1	1	$C^{5_15_2}$
H_d	$\mathbf{1}$	$\mathbf{2}$	-1/2	1	1	$C^{5_15_2}$
ν_{R2}^c	$\mathbf{1}$	$\mathbf{1}$	0	-3/2	0	$C_3^{5_1}$
ν_{R3}^c	$\mathbf{1}$	$\mathbf{1}$	0	-7/2	1	$C_3^{5_1}$
L_1	$\mathbf{1}$	$\mathbf{2}$	-1/2	-1/2	2	$C_3^{5_2}$
L_2	$\mathbf{1}$	$\mathbf{2}$	-1/2	-3/2	1	$C_3^{5_2}$
L_3	$\mathbf{1}$	$\mathbf{2}$	-1/2	-5/2	1	$C_3^{5_2}$
μ_R^c	$\mathbf{1}$	$\mathbf{1}$	1	3/2	1	$C^{5_15_2}$
τ_R^c	$\mathbf{1}$	$\mathbf{1}$	1	7/2	1	$C^{5_15_2}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	0	0	1	$C_3^{5_1}$
N	$\mathbf{1}$	$\mathbf{1}$	0	1	1	$C^{5_15_2}$
A	$\mathbf{1}$	$\mathbf{1}$	0	1	0	$C_3^{5_2}$
a	$\mathbf{1}$	$\mathbf{1}$	0	3	0	$C^{5_15_2}$
b	$\mathbf{1}$	$\mathbf{1}$	0	4	1	$C^{5_15_2}$
c	$\mathbf{1}$	$\mathbf{1}$	0	5	1	$C^{5_15_2}$
e	$\mathbf{1}$	$\mathbf{1}$	0	6	0	$C^{5_15_2}$
f	$\mathbf{1}$	$\mathbf{1}$	0	7	0	$C^{5_15_2}$
ψ	$\mathbf{1}$	$\mathbf{1}$	0	0	0	$C_3^{5_2}$

Table 1: Matter fields and flavons

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_R$	$\mathbb{Z}_3$	String State
χ_A	$\mathbf{1}$	$\mathbf{1}$	1	5/2	1	$C^{5_15_2}$
$\overline{\chi}_A$	$\mathbf{1}$	$\mathbf{1}$	-1	-1/2	2	$C^{5_15_2}$
χ_e	$\mathbf{1}$	$\mathbf{2}$	-1/2	9/2	1	$C^{5_15_2}$
$\overline{\chi}_e$	$\mathbf{1}$	$\mathbf{2}$	1/2	-5/2	2	$C^{5_15_2}$
χ_a	$\mathbf{1}$	$\mathbf{2}$	-1/2	5/2	2	$C^{5_15_2}$
$\overline{\chi}_a$	$\mathbf{1}$	$\mathbf{2}$	1/2	-1/2	1	$C^{5_15_2}$

Table 2: Messenger fields

where much larger means here larger by about a factor 5. Then, in this approximation, the couplings to ν^c_{R3} will lead to the neutrino mass eigenvalue $m_3 \simeq 0.05$ eV and the couplings to ν^c_{R2} to the smaller mass eigenvalue $m_2 \simeq 0.01$ eV. Clearly, since $m^{\mathrm{E}}_{\mathrm{LR}}$ is diagonal, the lepton mixing matrix $U_{\mathrm{MNS}} = R_{23} U_{13} R_{12}$ will be entirely given by the diagonalization matrix, i.e. $U_{\mathrm{MNS}} = U^\dagger_{\nu_{\mathrm{L}}}$ with $\mathrm{diag}(m_1, m_2, m_3) = U_{\nu_{\mathrm{L}}}\, m^\nu_{\mathrm{LR}}\, U^\dagger_{\nu_{\mathrm{R}}}$. Thus we find for the MNS mixings (assuming real flavon vevs for simplicity):[7]

$$
\begin{aligned}
\tan(\theta_{23}) &\approx \frac{\langle e \rangle}{\langle f \rangle}\,, && (13)\\
\tan(\theta_{12}) &\approx \frac{\langle a \rangle}{c_{23}\,\langle b \rangle - s_{23}\,\langle c \rangle}\,, && (14)\\
\theta_{13} &\approx 0\,, && (15)
\end{aligned}
$$

with m_3 and m_2 given by

$$
\begin{aligned}
m_3 &\approx \sqrt{\langle e \rangle^2 + \langle f \rangle^2}\,\frac{g_{5_1} v_{\mathrm{u}}}{\langle \psi \rangle} = \frac{\langle e \rangle}{s_{23}}\,\frac{g_{5_1} v_{\mathrm{u}}}{\langle \psi \rangle}\,, && (16)\\
m_2 &\approx \sqrt{\langle a \rangle^2 + (c_{23}\,\langle b \rangle - s_{23}\,\langle c \rangle)^2}\,\frac{g_{5_1} v_{\mathrm{u}}}{\langle \psi \rangle} = \frac{\langle a \rangle}{s_{12}}\,\frac{g_{5_1} v_{\mathrm{u}}}{\langle \psi \rangle}\,, && (17)\\
m_1 &\approx 0\,. && (18)
\end{aligned}
$$

We see that obtaining nearly maximal atmospheric mixing θ_{23} requires $\langle e \rangle \approx \langle f \rangle$. Obtaining a large (but non-maximal) solar mixing θ_{12} requires $\sqrt{2}\,\langle a \rangle \sim \langle b \rangle - \langle c \rangle$. Clearly, the neutrino data fixes $\langle e \rangle / \langle \psi \rangle$, $\langle f \rangle / \langle \psi \rangle$ (from m_3 and $\theta_{23} \approx \pi/4$), $\langle a \rangle / \langle \psi \rangle$ (from m_2) and the combination $(\langle b \rangle - \langle c \rangle)/\langle \psi \rangle$ from consistency with experimental data for θ_{12} (currently $\theta_{12} \approx 30^\circ$). We have neglected effects from the RG evolution of the neutrino parameters at this stage.

Finally, we remark that although the right-handed neutrinos $\nu^c_{\mathrm{R}i}$ as well as the inflaton ϕ are SM-singlets, we shall have in mind that at some stage, they transform in a representation of the gauge group G, a copy of which is associated with each of

[7] Since we assume a form of sequential dominance (SD) [9] for Y_ν, it is not surprising that the formulae for the mixing angles are very similar to the ones for see-saw neutrino masses under the assumption of SD.

the three stacks of D-branes.[8] G is just (spontaneously) broken completely on D5_1, whereas on D5_2 the SM gauge group $G_{321} \subset G$ is unbroken. To demonstrate this, let us consider two additional U(1) symmetries, $\mathrm{U}(1)_{5_1}$ on D5_1 and $\mathrm{U}(1)_{5_2}$ on D5_2, and assign charges to the SM-singlet fields $\phi, \nu^{\mathrm{c}}_{\mathrm{R}i}, N$ and ψ. First, giving H_{u} and H_{d} $\mathrm{U}(1)_{5_1}$-charge 1, we see that ϕ has charge -2 and thus N has charge 1. From the FN diagram in Fig. 3 we can determine the charges of the messenger fields if we assign a $\mathrm{U}(1)_{5_1}$-charge q to the right-handed neutrinos $\nu^{\mathrm{c}}_{\mathrm{R}i}$ and finally the flavons F_{ij}, which are intersection states $C^{5_1 5_2}$, end up with charge $-(q+1)$. Note that only fields which are assigned to string states $C_3^{5_1}$ and $C^{5_1 5_2}$ can be charged under $\mathrm{U}(1)_{5_1}$ and only fields $C_3^{5_2}$ and $C^{5_1 5_2}$ can be charged under $\mathrm{U}(1)_{5_2}$. Similarly for the $C_3^{5_2}$ state ψ, from the FN diagram in Fig. 3 we see how giving it a $\mathrm{U}(1)_{5_2}$-charge p determines e.g. the charge of $\bar{\chi}_{F_{ij}}$ to be $-p$ and the charge of the flavons F_{ij} to be p. It is easy to see that this charge assignment can be extended consistently to all the fields of the model.

4 A brief review of the hybrid inflation model

We will now review the inflation model [5] where the required small couplings might also originate from the string motivated scenario outlined here [6]. In fact, the requirement of very small couplings - and their possible common solution - might provide a link between the origin of (Dirac) neutrino masses and the physics driving inflation.

The starting point of the field theory model is the following part of the superpotential Eq. (10) relevant to inflation (with $\lambda \sim \kappa \sim g_{5_1} = 10^{-10}$) and the corresponding soft terms:

$$W_{inf.} = \lambda \phi H_{\mathrm{u}} H_{\mathrm{d}} + \kappa \phi N^2. \tag{19}$$

[8] This is one point which distinguishes our approach from approaches where right-handed neutrinos are gauge singlets and propagate in the bulk.

$$V_{soft} = V(0) + \lambda A_\lambda \phi H_{\rm u} H_{\rm d} + \kappa A_\kappa \phi N^2 + h.c.$$
$$+ m_0^2(|H_{\rm u}|^2 + |H_{\rm d}|^2 + |N|^2) + m_\phi^2 |\phi|^2 \, . \tag{20}$$

ϕ and N are, respectively, the inflaton and waterfall fields. These fields are singlets of the Minimal Supersymmetric Standard Model (MSSM) [10] gauge group and the other fields are just the usual quarks and Higgs multiplets of the MSSM with standard MSSM quantum numbers.

This model utilises hybrid inflation [11, 12, 13, 14, 15] to provide a simultaneous solution to the strong CP and μ [10] problems, as we will now briefly outline (for a more detailed discussion see [5] and the references therein):

- During inflation, the scalar potential reduces to the simple form

$$V = V(0) - m_\phi^2 \phi^2 \, , \tag{21}$$

  and the vacuum energy dominates the potential during inflation. Inflation ends by a second order phase transition if ϕ reaches a critical value

$$\phi_c^\pm = \frac{A_k}{4k}\left(-1 \pm \sqrt{1 - 4\frac{m_N^2}{A_k^2}}\right) . \tag{22}$$

  In the minimum of the potential after inflation, the field values are [5]

$$\langle\phi\rangle = -\frac{A_\lambda}{4\lambda} \, , \tag{23}$$
$$\langle N\rangle = \frac{A_\lambda}{2\sqrt{2}\lambda}\sqrt{1 - 4\frac{m_0^2}{A_\lambda^2}} \, . \tag{24}$$

- The model has the Peccei-Quinn (PQ) symmetry $\mathrm{U(1)_{PQ}}$ where the global charges of the fields satisfy

$$Q_\phi + Q_{H_{\rm u}} + Q_{H_{\rm d}} = 0 \, , \quad Q_\phi + 2Q_N = 0 \, . \tag{25}$$

- After inflation has ended the VEV of the inflaton $\langle\phi\rangle$ both generates the μ term (in a similar way to the NMSSM [16, 17]), when $\lambda\phi H_{\rm u} H_{\rm d} \to \mu H_{\rm u} H_{\rm d}$ and breaks the $\mathrm{U(1)_{PQ}}$ [18] symmetry solving the strong CP problem.

The model leads to the following constraints on the scales and couplings of the theory: With the VEV of ϕ given by $\langle\phi\rangle = -\frac{A_\lambda}{4\lambda}$, we can use the constraints on the axion decay constant to determine the value of λ. For soft terms at the TeV scale, we see that an axion decay constant should lie in the range 10^{10} GeV $\leq f_a \leq 10^{13}$ GeV (see [19, 20] for derivation of the allowed region) requires that λ lie in the range $10^{-7} \geq \lambda \geq 10^{-10}$. If we take the smallest value in this range this allows[9] $\lambda \sim 10^{-10}$ and it is this small coupling that we have used in the neutrino sector.

Requiring the stability of the vacuum post inflation and that inflation ends imposes constraints on the range of allowed ratios of the soft terms:

$$8m_0^2 > |A_\lambda|^2 > 4m_0^2 \ . \tag{26}$$

Hence we were able to show that:

$$\mu^2 = (0.25 - 0.5)m_0^2 \ , \tag{27}$$

where m_0 is a soft scalar mass common to many of the matter fields of order a TeV, whose universality was shown to result from the string construction.

5 Conclusions

We have shown that Type I string theory provides a natural framework for the construction of Dirac neutrino models. Small couplings are essential to our model and Type I string theory, compactified on an orbifold, is equipped with a geometric explanation for these small couplings. The large ratios between the different couplings are explained in terms of the anisotropy of the compact space. These extra dimensions are not especially large since two radii are of order the Planck length and one approximately 10^{-8} GeV^{-1}.

[9]The origin of this small coupling is discussed in Appendix A

These small Yukawa couplings allow us to relate physics at very different scales: namely the neutrino mass, electroweak and Peccei-Quinn scales. In so doing we connect the physics of neutrino mass and inflation. Specifically we construct a model which consistently describes neutrino mass generation and an inflationary solution to the strong CP and μ problems and has its roots in Type I string theory. Consistency with the measured values of the MNS matrix is achieved by generating the mass matrix from non-renormalisable operators. These operators are field theoretic in origin, coming from a supersymmetric generalisation of the Froggatt-Nielsen mechanism. The set of allowed operators is restricted by the inclusion of an additional $\mathrm{U}(1)_{\mathrm{R}} \times \mathbb{Z}_3$ symmetry which leads to the angles and masses shown in Eq.(14-18). It would be interesting to extend this model to include the quark sector. In such an approach it may be possible to relate the Cabibbo angle θ_{C} to the neutrino mass hierarchy m_2/m_3 in terms of an expansion parameter $\lambda = \theta_{\mathrm{C}}$ in a more direct way than in the see-saw mechanism.

The Dirac nature of neutrino masses would have far reaching phenomenological consequences: Dirac neutrinos would, for instance, not induce any signal in neutrinoless double beta ($0\nu\beta\beta$) decay experiments [27]. Since in our scenario the Dirac mass matrix is not directly related to the mass matrices of quarks and charged leptons, there is *a priori* no reason for favouring a hierarchical neutrino mass spectrum compared to an inverted or quasi-degenerate one. Although we have discussed the case of a hierarchical spectrum in this work, from a model building point of view a quasi-degenerate neutrino mass spectrum for Dirac neutrinos can emerge, e.g., from additional Abelian, non-Abelian or discrete symmetries in our scenario. Such quasi-degenerate Dirac neutrino masses could be observable in Tritium β-decay experiments like KATRIN [28]. Together with non-observation of $0\nu\beta\beta$ decay, this could prove the Dirac nature of neutrinos. In our scenario, the inflaton can only have small couplings of order $g_{5_1} \sim 10^{-10}$ to matter. This generically implies a low reheat temperature

after inflation $\mathcal{O}(1-10)$ GeV, which is interesting with respect to gravitino (and other similar) constraints in some supergravity models [29]. On the other hand, with Dirac neutrinos the original leptogenesis mechanism [30] via the out-of equilibrium decay of heavy right-handed neutrinos is obviously not available. However other versions of leptogenesis, such as Dirac leptogenesis [31], which rely on sphaleron transitions for converting lepton into baryon asymmetry could work for a low reheat temperature [32] due to preheating. In summary, we feel that in addition to interesting theoretical issues regarding, e.g., the embedding in string theory, scenarios of Dirac neutrinos and inflation like the one discussed in this work have a rich phenomenology, which deserves further exploration.

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Appendix

A String Selection Rules

We will now review the properties of Type I string theory relevant for model building, first presented in [21], and summarise the model presented in [6]. We will be working with a D-brane setup which includes a geometric mechanism for generating small gauge and Yukawa couplings. We consider the class of spaces known as orientifolds (see [22] for a study of possible orientifolds) requiring the addition of intersecting stacks of orthogonal D5-branes and space-filling D9-branes for consistency. These spaces are all constructed from a 6-torus and it is the volume and anisotropy of this torus that leads to the generation of a hierarchy of couplings. The 6-torus itself is constructed out of three 2-tori each of which has one radius associated with it. We will show that if one radius is of order 10^{-8} GeV^{-1} and the other two are of order 10^{-18} GeV^{-1} then we obtain a coupling of order 10^{-10}.

After compactification we end up with, in the most general case, a model consisting of three orthogonal stacks of D5-branes and a stack of D9-branes.

The gauge couplings on the branes can be shown to be the following functions of the extra-dimensional radii.

$$g_{5_i}^2 = \frac{2\pi\lambda_{\mathrm{I}}}{R_i^2 M_*^2} \tag{A.1}$$

$$g_9^2 = \frac{2\pi\lambda_{\mathrm{I}}}{R_1^2 R_2^2 R_3^3 M_*^2} \tag{A.2}$$

Where λ_{I} is the ten dimensional dilaton that governs the strength of string interactions, M_* is the Type I string scale and R_i are the radii of compactification.

The non-canonical, $D = 4$, $\mathcal{N} = 1$ effective superpotential has only $\mathcal{O}(1)$ Yukawa couplings [23], but the Kähler metric, although diagonal, is significantly different

from the identity. To understand our theory in the low energy, after the dilaton and moduli have acquired vevs, we must canonically normalise the Kähler potential and take the flat limit in which $M_p \to \infty$ while $m_{3/2}$ is kept constant [24, 25]. This gives a theory containing superfields with canonical kinetic terms interacting via renormalisable operators. Notice that the Yukawa couplings can be identified with the gauge couplings (up to the $\mathcal{O}(1)$ factors present before normalisation):

$$W = g_9 \left(C_1^9 C_2^9 C_3^9 + C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1} + \sum_{i=1}^{3} C_i^9 C^{95_i} C^{95_i} \right) + \sum_{i,j,k=1}^{3} g_{5_i} \big(C_1^{5_i} C_2^{5_i} C_3^{5_i} + C_i^{5_i} C^{95_i} C^{95_i} + d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} + \frac{1}{2} d_{ijk} C^{5_j 5_k} C^{95_j} C^{95_k} \big). \tag{A.3}$$

where the C terms are low energy excitations of strings: charged chiral superfields. The superscripts denote the branes which the strings end on and terms with different subscripts transform differently under the gauge group associated with the brane.

The $D = 4$ Planck scale is related to the string scale by

$$M_p^2 = \frac{8 M_*^8 R_1^2 R_2^2 R_3^2}{\lambda_{\mathrm{I}}^2}. \tag{A.4}$$

From this and Eqs. (A.1) to (A.2) we find that

$$g_{5_1} g_{5_2} g_{5_3} g_9 = 32\pi^2 \left(\frac{M_*}{M_p} \right)^2 . \tag{A.5}$$

Another important relation is

$$\lambda_{\mathrm{I}} = \frac{g_{5_1} g_{5_2} g_{5_3}}{2\pi g_9} . \tag{A.6}$$

In the model discussed in the next section, we shall require at least one coupling of $\mathcal{O}(10^{-10})$ and one of $\mathcal{O}(1)$. According to the above results, this constrains the size of our radii and the value of the string scale. For definiteness we consider the case where $g_{5_1} \sim 10^{-10}$ and the remaining gauge couplings are all $\mathcal{O}(1)$. From Eq. (A.5) we see this is clearly allowed if we have a 10^{13} GeV string scale. Specifically our couplings are $g_{5_2} = \sqrt{\frac{4\pi}{24}}$ (to give $\alpha_{\mathrm{GUT}} = 1/24$, consistent with gauge coupling unification), $g_{5_3} = g_9 = 2$ and $g_{5_1} = 10^{-10}$ gives $M_* = 10^{13}$ GeV.

The hierarchy in gauge couplings corresponds to a hierarchy in the radii. Using Eqs. (A.6) and (A.1) for the above couplings we find that

$$R_1^{-1} = 1.3 \times 10^8 \text{ GeV} \tag{A.7}$$

$$R_2^{-1} = 9.1 \times 10^{17} \text{ GeV} \tag{A.8}$$

$$R_3^{-1} = 2.4 \times 10^{18} \text{ GeV}. \tag{A.9}$$

These radii are all too small to have Kaluza-Klein (KK) or winding modes that will be readily excitable at collider energies. The winding modes of R_1 are $\approx n10^{18}$ GeV and R_2 and R_3 have winding modes of $\approx n10^8$ GeV. The KK modes for R_1 are $\approx n10^8$ GeV and R_2 and R_3 are $\approx n10^{18}$ GeV. In principle these massive modes could affect inflation. However the inflationary scale is 10^8 GeV so it is unlikely that these modes would appear with any great abundance.

A.1 Methodology

Our approach in this paper is one of string inspired phenomenology. We make use of a number of the generic properties of low energy effective string theory so as to keep our analysis as general as possible and avoid specialising to a particular model. The rules we enforce are:

- All supersymmetric terms must be found within the low energy effective superpotential Eq. (3).
- The string states, $C^{5_1 5_2}$ etc., can represent more than one low energy field.
- Each low energy field can only be assigned to one string state.
- The gauge quantum numbers of a string state are determined by the stacks of branes which it ends on.

We will now clarify and expand on the final point. Clearly the ends of the string can either both be on the same stack of branes or attached to two different stacks. The string ends' locations determine the transformation properties of their low energy excitations since the brane stacks have an associated gauge group under which the strings transform. If both string ends attach to the same brane stack then it is commonly the case that fields transform as reducible representations of that stack's gauge group, typically U(N). In the other case, with ends on different stacks, then the fields generally transform as fundamental representations of both stack's gauge groups. We impose this requirement in our model building.

B Our Convention for Lepton Mixing

For the mass matrix of the charged leptons $m^{\mathrm{E}}_{\mathrm{LR}} = Y_{\mathrm{e}} v_{\mathrm{d}}$ defined by $\mathcal{L}_{\mathrm{e}} = -m^{\mathrm{E}}_{\mathrm{LR}} \overline{e}^{f}_{\mathrm{L}} e^{\mathrm{c}f}_{\mathrm{R}}$ + h.c. and for the Dirac neutrino mass matrix $m^{\nu}_{\mathrm{LR}} = Y_{\nu} v_{\mathrm{u}}$ defined by $\mathcal{L}_{\nu} = -m^{\nu}_{\mathrm{LR}} \overline{\nu}^{f}_{\mathrm{L}} \nu^{\mathrm{c}f}_{\mathrm{R}}$ + h.c., where $v_{\mathrm{u}} = \langle H^0_{\mathrm{u}} \rangle$ and $v_{\mathrm{d}} = \langle H^0_{\mathrm{d}} \rangle$, the change from flavour basis to mass eigenbasis can be performed with the unitary diagonalization matrices $U_{e_{\mathrm{L}}}, U_{e_{\mathrm{R}}}$ and $U_{\nu_{\mathrm{L}}}, U_{\nu_{\mathrm{R}}}$ by

$$U_{e_{\mathrm{L}}}\, m^{\mathrm{E}}_{\mathrm{LR}}\, U^{\dagger}_{e_{\mathrm{R}}} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_{\nu_{\mathrm{L}}}\, m^{\nu}_{\mathrm{LR}}\, U^{\dagger}_{\nu_{\mathrm{R}}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{B.10}$$

The mixing matrix in the lepton sector, the MNS matrix, is then given by

$$U_{\mathrm{MNS}} = U_{e_{\mathrm{L}}} U^{\dagger}_{\nu_{\mathrm{L}}}\ . \tag{B.11}$$

We use the parameterisation $U_{\mathrm{MNS}} = R_{23} U_{13} R_{12}$ with R_{23}, U_{13}, R_{12} defined as

$$R_{12} := \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{13} := \begin{pmatrix} c_{13} & 0 & \widetilde{s}_{13} \\ 0 & 1 & 0 \\ -\widetilde{s}^{*}_{13} & 0 & c_{13} \end{pmatrix}, \quad R_{23} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

and where s_{ij} and c_{ij} stand for $\sin(\theta_{ij})$ and $\cos(\theta_{ij})$, respectively. δ is the Dirac CP phase relevant for neutrino oscillations and we have defined $\widetilde{s}_{13} := s_{13} e^{-i\delta}$.

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